SYDNEY GRAMMAR SCHOOL

CANDIDATE NUMBER

2021 Trial Examination

Form VI Mathematics Advanced

Friday 20th August 2021
8:40am

General Instructions

- Reading time — 10 minutes
- Working time — 3 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.

Total Marks: 100

Section I (10 marks) Questions 1 – 10
- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.
- Write your candidate number on each page.

Section II (90 marks) Questions 11 – 33
- Because of the nature of this task, greater weight than normal will be placed on working. Clear reasoning and full calculations are required.
- Answer the questions in this paper in the spaces provided.
- Write your candidate number on each page.
- This section is divided in four parts. Extra writing paper is provided at the end of each part.

Your sheets must be ORDERED
then scanned and uploaded in a
SINGLE PDF FILE
to the Schoology page of your mathematics class

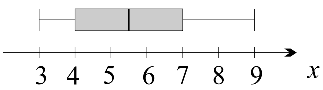
Checklist
- Reference sheet
- Multiple-choice answer sheet
- Candidature: 101 pupils

Writer: LYL

Section I

Questions in this section are multiple-choice.
Choose the single best answer for each question and record it on the provided answer sheet.

1.



What is the interquartile range for the box-and-whisker plot above?

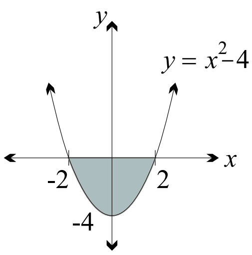
(A) $1{\cdot}0$

(B) $1{\cdot}5$

(C) $3{\cdot}0$

(D) $6{\cdot}0$

2.



In the diagram above, what is the area of the shaded region?

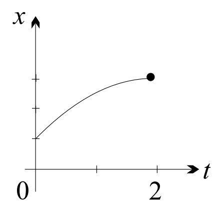
(A) $\frac{16}{3}$ units^2

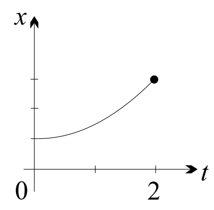
(B) 8 units^2

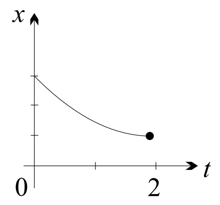
(C) $\frac{32}{3}$ units^2

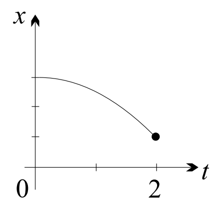
(D) $\frac{64}{3}$ units^2

3. A particle is moving in a straight line. For $0 \leq t \leq 2$, its velocity is positive and its acceleration is negative. Which graph best represents the displacement function $x(t)$ of this particle?

(A) 

(C) 

(B) 

(D) 

4. What is the limiting sum of the series $\frac{13}{6}, \frac{13}{36}, \frac{13}{216}, \ldots$?

(A) $\frac{65}{36}$

(B) $\frac{13}{7}$

(C) $\frac{13}{5}$

(D) $\frac{14}{3}$

5. Which expression is equal to $\displaystyle\int \frac{4}{1-\sin^2 4x}\, dx$?

(A) $\frac{1}{4}\tan 4x + C$

(B) $4\tan^2 4x + C$

(C) $\tan 4x + C$

(D) $\frac{1}{4}\tan^2 4x + x + C$

6. What is the derivative of $\dfrac{e^{2-x}}{x^2}$?

(A) $\dfrac{xe^{2-x} - 2e^{2-x}}{x^3}$

(B) $\dfrac{-2e^{2-x} - xe^{2-x}}{x^3}$

(C) $\dfrac{xe^{2-x} + 2e^{2-x}}{x^3}$

(D) $\dfrac{2e^{2-x} - xe^{2-x}}{x^3}$

7. A function is defined by the rule

$$f(x) = \begin{cases} 1 & \text{for } x < 1 \\ x + 2 & \text{for } x \geq 1 \end{cases}$$

Which statement is incorrect?

(A) The value of $f(-2)$ is 1.

(B) The graph is not continuous at $x = 1$.

(C) The domain is all real values for x.

(D) The range is $f(x) \geq 1$.

8. What is the domain of the function $f(x) = \dfrac{1}{\sqrt{x^2 - 9}}$?

(A) $(-\infty, -3) \cup (3, \infty)$

(B) $(-\infty, -3)$

(C) $[-\infty, -3] \cup [3, \infty]$

(D) $(3, -3)$

9. Using the trapezoidal rule with 4 subintervals, which expression gives the best approximation of the area under the curve $y = xe^{2x}$ between $x = 1$ and $x = 2$?

(A) $\frac{1}{8}(e^2 + 1{\cdot}25e^{2{\cdot}5} + 1{\cdot}5e^3 + 1{\cdot}75e^{3{\cdot}5} + e^4)$

(B) $\frac{1}{8}(e^2 + 2{\cdot}5e^{2{\cdot}5} + 3e^3 + 3{\cdot}5e^{3{\cdot}5} + 2e^4)$

(C) $\frac{1}{4}(e^2 + 1{\cdot}25e^{2{\cdot}5} + 1{\cdot}5e^3 + 1{\cdot}75e^{3{\cdot}5} + e^4)$

(D) $\frac{1}{4}(e^2 + 2{\cdot}5e^{2{\cdot}5} + 3e^3 + 3{\cdot}5e^{3{\cdot}5} + 2e^4)$

10. What is the nature and coordinates of the stationary point of the curve $y = \dfrac{\ln x}{x^3}$?

(A) A minimum turning point at $\left(e^{\frac{1}{3}}, \frac{1}{3e}\right)$.

(B) A maximum turning point at $\left(\frac{1}{3e}, e^{\frac{1}{3}}\right)$.

(C) A minimum turning point at $\left(\frac{1}{3e}, e^{\frac{1}{3}}\right)$.

(D) A maximum turning point at $\left(e^{\frac{1}{3}}, \frac{1}{3e}\right)$.

End of Section I

The paper continues in the next section

QUESTION ELEVEN (2 marks) **Marks**

Find the equation of the tangent to the curve $y = x^3 - x + 4$ at $x = 1$. $\boxed{2}$

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QUESTION TWELVE (4 marks) **Marks**

Differentiate:

(a) $y = \sqrt{x}$ $\boxed{1}$

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(b) $y = \cos 2x$ $\boxed{1}$

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(c) $y = x^3 \ln x$ $\boxed{2}$

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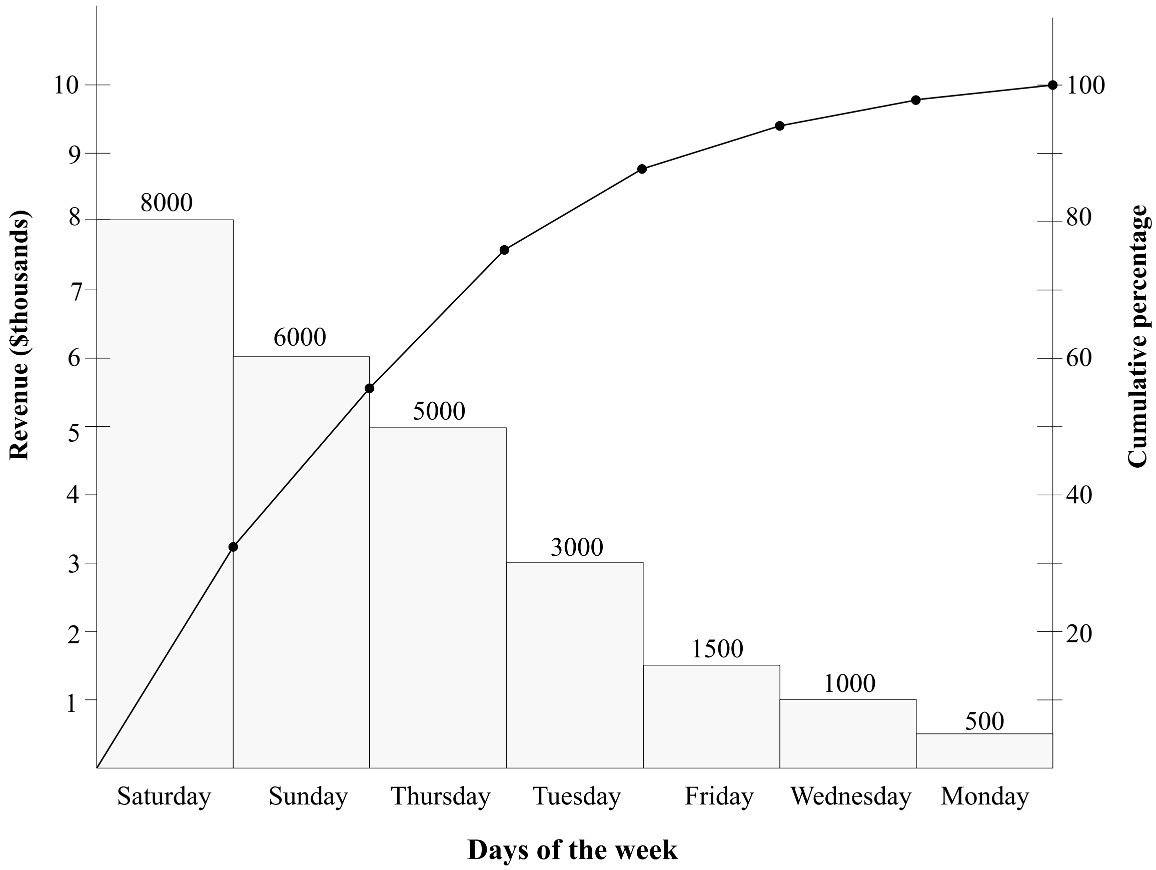
QUESTION THIRTEEN (3 marks) **Marks**



The diagram above shows a Pareto chart of the revenue that a bookshop made during a week.

(a) What percentage of the total revenue was made on the week days? 2

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(b) Suggest one valid action the manager may consider using the Pareto chart results. 1

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QUESTION FOURTEEN (2 marks) **Marks**

If $\cos\theta = -\frac{3}{7}$ and $\tan\theta$ is positive, find the value of $\sin\theta$. Leave your answer in simplified form. $\boxed{2}$

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QUESTION FIFTEEN (4 marks) **Marks**

The graph of $y = \dfrac{2}{x}$ is translated upwards by 1 unit followed by a reflection in the x-axis.

(a) State the equation of the new graph. $\boxed{1}$

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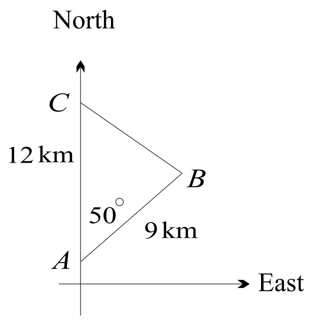
(b) Sketch the new graph. Clearly indicate any intercepts with the axes and any asymptotes. $\boxed{3}$

QUESTION SIXTEEN (4 marks)

Marks

How many terms are there in the series $11 + 13 + 15 + \cdots$ if the sum is 375? $\boxed{2}$

QUESTION SEVENTEEN (3 marks) **Marks**



The diagram above shows three checkpoints A, B and C in an orienteering event. Checkpoints A and C are such that C is 12 km due north of A. One participant starts from A and walks in the direction of 050° T. After 9 km the participant arrives at checkpoint B.

(a) Find the distance BC. Give your answer correct to 3 significant figures. 1

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(b) What is the true bearing of C from B? Give your answer correct to the nearest degree. 2

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QUESTION EIGHTEEN (2 marks) **Marks**

Solve the following equation for x in terms of a: $\boxed{2}$

$$3\log_a x + 4 = 5\log_a x$$

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The paper continues on page 15.

QUESTION NINETEEN (3 marks) **Marks**

(a) Find $\displaystyle\int \frac{5}{x}\,dx$. $\boxed{1}$

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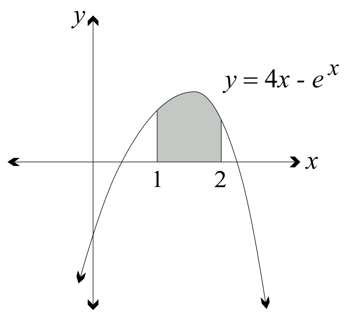
(b) $\boxed{2}$



For the diagram above, find the exact area bounded by the curve, $y = 4x - e^x$ and the x-axis between $x = 1$ and $x = 2$.

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QUESTION TWENTY (5 marks) **Marks**

x	1	2	3	4
$P(X = x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

The table above shows the probability distribution of a spinner for a board game.
Let X be the outcome of the spinner.

(a) Find $P(X \leq 3)$. 1

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(b) Find the expected value $E(X)$. 2

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(c) Find the variance $\text{Var}(X)$. 2

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QUESTION TWENTY-ONE (7 marks) **Marks**

Consider the curve $y = 4x^2 - 2x^3$.

(a) Find the stationary points of the curve $y = 4x^2 - 2x^3$. Determine their nature. $\boxed{3}$

(b) Show that there is an inflection point at $\left(\frac{2}{3}, \frac{32}{27}\right)$ on the curve. $\boxed{1}$

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(c) For what interval is the curve $y = 4x^2 - 2x^3$ increasing? $\boxed{1}$

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(d) Hence, sketch the graph of the curve $y = 4x^2 - 2x^3$. Clearly label the stationary points, the point of inflection and any intercepts with the axes. $\boxed{2}$

QUESTION TWENTY-TWO (4 marks) **Marks**

A particle moves along a straight line so that its displacement x metres to the right of a fixed point O is given by

$$x = 12\ln(t+2) - 2t + 5\,,$$

where the time t is measured in seconds.

(a) What is the initial position of the particle? Give your answer in exact form. $\boxed{1}$

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(b) Find the expression for the velocity of the particle at time t. $\boxed{1}$

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(c) Find the time when the particle is at rest. $\boxed{1}$

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(d) What happens to the acceleration eventually? $\boxed{1}$

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This is the halfway point of the whole paper.

QUESTION TWENTY-THREE (2 marks)

Marks

2

A curve $y = f(x)$ passes through $\left(\dfrac{\pi}{2}, \dfrac{-\pi}{2}\right)$ and has the gradient function

$$\frac{dy}{dx} = 4\cos 2x + 1\,.$$

Find the equation of the curve.

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QUESTION TWENTY-FOUR (3 marks)

Marks

Solve the equation $2\sin 2x = 1$ for $0 \le x \le 2\pi$.

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QUESTION TWENTY-FIVE (8 marks) **Marks**

The table below shows the latitude (degrees south of the equator) and the temperature on a particular day (°C) of eight locations in Australia.

Location	Latitude	Temperature
	°South	°C
Alice Springs	24	28
Byron Bay	29	28
Carnavon	25	32
Geraldton	29	29
Hobart	43	17
Mt. Isa	21	34
Port Lincoln	35	21
Wagga Wagga	35	23

(a) Draw a scatter plot to show any potential relationship between latitude and temperature in the southern hemisphere. Let the horizontal axis be latitude and the vertical axis be temperature. 2

(b) Describe the relationship between latitude and temperature observed in the scatter plot. [1]

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(c) By eye, estimate and draw in the line of best fit on your scatter plot in part (a).
Hence determine the y-intercept and the gradient for your line.
Give your answers to 2 decimal places. [2]

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(d) (i) Using your calculator, find Pearson's correlation coefficient r for this data.
Give your answer to 4 significant figures. [1]

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(ii) Comment on the significance of the value r for this set of data. [1]

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(e) Using your calculator, find the equation of the line of regression. Give the y-intercept and the gradient for the line to 3 significant figures. [1]

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QUESTION TWENTY-SIX (4 marks) **Marks**

(a) Graph $y = \sin x$ for $0 \leq x \leq 2\pi$. $\boxed{2}$

(b) Shade the regions bounded by the curve $y = \sin x$, the x-axis and between $x = 0$ and $x = \dfrac{3\pi}{2}$. Calculate the total area of these regions. $\boxed{2}$

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QUESTION TWENTY-SEVEN (3 marks) **Marks**

Evaluate:

(a) $\displaystyle\int \frac{x}{x^2-5}\,dx$ $\boxed{1}$

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(b) $\displaystyle\int_1^5 \frac{x^2+3}{x}\,dx$ $\boxed{2}$

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QUESTION TWENTY-EIGHT (4 marks) **Marks**

Hugo has three years to save $25 000 for a holiday.

(a) Hugo deposits a single lump sum into an account paying 8% p.a. interest compounded every 6 months. What lump sum is needed to ensure he can afford his holiday in three years time? Give your answer to the nearest dollar. $\boxed{2}$

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(b) $\boxed{2}$

Periods	Interest rate per period					
n	3%	4%	5%	6%	8%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0300	2.0400	2.0500	2.0600	2.0800	2.1200
3	3.0909	3.1216	3.1525	3.1836	3.2464	3.3744
4	4.1836	4.2465	4.3101	4.3746	4.5061	4.7793
5	5.3091	5.4163	5.5256	5.6371	5.8666	6.3528
6	6.4684	6.6330	6.8019	6.9753	7.3359	8.1152
7	7.6625	7.8983	8.1420	8.3938	8.9228	10.0890
8	8.8923	9.2142	9.5491	9.8975	10.6366	12.2997
9	10.1591	10.5828	11.0266	11.4913	12.4876	14.7757
10	11.4639	12.0061	12.5779	13.1808	14.4866	17.5487
11	12.8078	13.4864	14.2068	14.9716	16.6455	20.6546
12	14.1920	15.0258	15.9171	16.8699	18.9771	24.1331

Hugo instead decides to make regular deposits to an annuity to save for his holiday.
He deposits $1800 at the end of each quarter over 3 years at 12% p.a. interest compounded quarterly.

Use the future value table above to determine if Hugo will have enough money to take his holiday. Show working to explain your answer.

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QUESTION TWENTY-NINE (6 marks) **Marks**

A closed cylindrical tank has a circular base of radius r metres and a height of h metres. It can hold $160\pi\,\text{m}^3$ of wheat. The material for the flat circular top and bottom costs \$10 per square metre and the material for the curved surface costs \$8 per square metre.

(a) Show that the height of the tank is $h = \dfrac{160}{r^2}$. $\boxed{1}$

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(b) Find the dimensions of the tank that minimise the cost of construction. $\boxed{5}$

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QUESTION THIRTY (3 marks) **Marks**

Prove the identity $(\sec\theta + \tan\theta)^2 = \dfrac{1+\sin\theta}{1-\sin\theta}$. $\boxed{3}$

QUESTION THIRTY-ONE (7 marks) **Marks**

Tom invented a dice game to play on his own. He throws a pair of six-sided dice repeatedly until the difference between the dice is 2 or 3.
If the difference is 2, Tom wins and the game ends.
If the difference is 3, Tom loses and the game ends.
If the difference is any other number, he continues to throw until the difference is a 2 or 3.

(a) Show that the probability that Tom wins on his first throw of the dice is $\dfrac{2}{9}$. $\boxed{2}$

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(b) Calculate the probability that the game continues to a second throw. $\boxed{1}$

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(c) What is the probability that Tom wins in one of the first three throws? Leave your answer in unsimplified form. $\boxed{2}$

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(d) Calculate the probability that Tom wins the game. $\boxed{2}$

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QUESTION THIRTY-TWO (6 marks) **Marks**

On the 1st January 2021, Amanda invested \$7000 into a bank account that paid interest at a rate of 6% p.a. compounded annually. Amanda decided to add \$700 to her account on the 1st January each year, beginning in 2022.

Let A_n be the amount in the account on the 1st January after n years, after interest and her deposit has been paid.

(a) Show that $A_2 = A_1 \times 1{\cdot}06 + 700$. 1

(b) Hence, determine how much was in her account on 1st January 2031, after interest and her deposit has been paid. Give your answer to the nearest dollar. 3

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(c) Amanda's friend, Bard, invested \$7000 into an account in a different bank on the 1st January 2021 and made no further payments. On 1st January 2031, Bard's balance was \$23 417. $\boxed{2}$

Calculate the annual rate of compound interest paid on Bard's account. Give your answer to 4 significant figures.

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The paper continues on the next page.

QUESTION THIRTY-THREE (3 marks) Marks

Find the area bounded by the curves $y = \sqrt{4x+8}$ and $5y - 2x = 12$. $\boxed{3}$

END OF PAPER

SYDNEY GRAMMAR SCHOOL

19/8/2021

SOLUTIONS

CANDIDATE NUMBER

2021 Trial Examination

Form VI Mathematics Advanced

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8:40am

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Checklist
- Reference sheet
- Multiple-choice answer sheet
- Candidature: 101 pupils

Writer: LYL

	Marks
Multiple Choice	10
Part A	22
Part B	24
Part C	25
Part D	19
TOTAL	

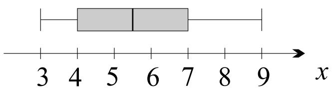
100

Section I

Questions in this section are multiple-choice.
Choose the single best answer for each question and record it on the provided answer sheet.

Q1	C
Q2	C
Q3	A
Q4	C
Q5	C
Q6	B
Q7	D
Q8	A
Q9	B
Q10	D

1.



What is the interquartile range for the box-and-whisker plot above?

(A) 1·0

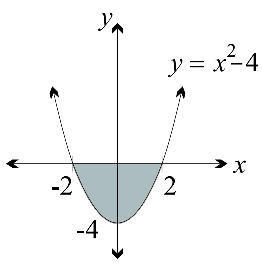
(B) 1·5

(C) 3·0

(D) 6·0

IQR $= 7 - 4$
$= 3$

2.



In the diagram above, what is the area of the shaded region?

(A) $\frac{16}{3}$ units^2

(B) 8 units^2

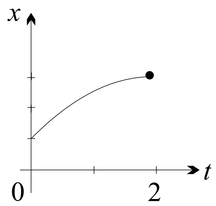
(C) $\frac{32}{3}$ units^2

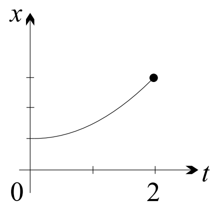
(D) $\frac{64}{3}$ units^2

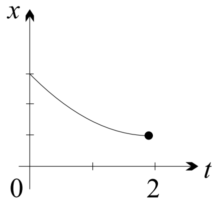
$$\int_0^2 (x^2 - 4)\,dx$$
$$= \left[\frac{x^3}{3} - 4x\right]_0^2$$
$$= \left(\frac{8}{3} - 8\right) - (0 - 0)$$
$$= \frac{8 - 24}{3}$$
$$= -\frac{16}{3}$$

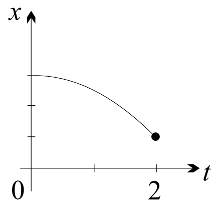
$A = 2 \times \frac{16}{3}$
$= \frac{32}{3}$ units^2.

3. A particle is moving in a straight line. For $0 \le t \le 2$, its velocity is positive and its acceleration is negative. Which graph best represents the displacement function $x(t)$ of this particle?

(A)


(C)


(B)


(D)


4. What is the limiting sum of the series $\frac{13}{6}, \frac{13}{36}, \frac{13}{216}, \ldots$?

(A) $\frac{65}{36}$

(B) $\frac{13}{7}$

(C) $\frac{13}{5}$

(D) $\frac{14}{3}$

$a = \frac{13}{6}$ $\quad r = \frac{1}{6}$

$S_\infty = \frac{13}{6} \div \frac{5}{6}$

$= \frac{13}{5}$

5. Which expression is equal to $\displaystyle\int \frac{4}{1 - \sin^2 4x}\, dx$?

(A) $\frac{1}{4}\tan 4x + C$

(B) $4\tan^2 4x + C$

(C) $\tan 4x + C$

(D) $\frac{1}{4}\tan^2 4x + x + C$

$\int \frac{4}{\cos^2 4x}\, dx$

$= \int 4 \sec^2(4x)\, dx$

$= 4 \times \frac{1}{4} \tan 4x + C$

6. What is the derivative of $\dfrac{e^{2-x}}{x^2}$?

(A) $\dfrac{xe^{2-x} - 2e^{2-x}}{x^3}$

(B) $\dfrac{-2e^{2-x} - xe^{2-x}}{x^3}$

(C) $\dfrac{xe^{2-x} + 2e^{2-x}}{x^3}$

(D) $\dfrac{2e^{2-x} - xe^{2-x}}{x^3}$

$u = e^{2-x}$ $\quad v = x^2$

$u' = -e^{2-x}$ $\quad v' = 2x$

$y' = \dfrac{-x^2 e^{2-x} - 2xe^{2-x}}{x^4}$

$= \dfrac{-\cancel{x}e^{2-x}(x+2)}{x^{\cancel{4}3}}$

$= \dfrac{-e^{2-x}(x+2)}{x^3}$

7. A function is defined by the rule

$$f(x) = \begin{cases} 1 & \text{for } x < 1 \\ x + 2 & \text{for } x \geq 1 \end{cases}$$

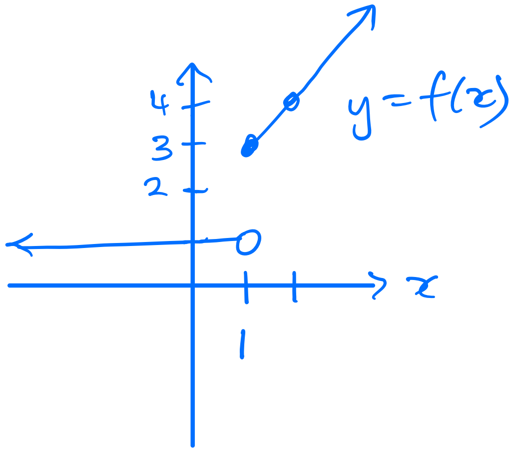
Which statement is incorrect?

(A) The value of $f(-2)$ is 1.

(B) The graph is not continuous at $x = 1$.

(C) The domain is all real values for x.

(D) The range is $f(x) \geq 1$.



8. What is the domain of the function $f(x) = \dfrac{1}{\sqrt{x^2 - 9}}$?

(A) $(-\infty, -3) \cup (3, \infty)$

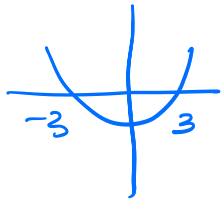
(B) $(-\infty, -3)$

(C) $[-\infty, -3] \cup [3, \infty]$

(D) $(3, -3)$

$x^2 - 9 > 0$

$x < -3 \cup x > 3$



9. Using the trapezoidal rule with 4 subintervals, which expression gives the best approximation of the area under the curve $y = xe^{2x}$ between $x = 1$ and $x = 2$?

(A) $\frac{1}{8}(e^2 + 1{\cdot}25e^{2{\cdot}5} + 1{\cdot}5e^3 + 1{\cdot}75e^{3{\cdot}5} + e^4)$

(B) $\frac{1}{8}(e^2 + 2{\cdot}5e^{2{\cdot}5} + 3e^3 + 3{\cdot}5e^{3{\cdot}5} + 2e^4)$

(C) $\frac{1}{4}(e^2 + 1{\cdot}25e^{2{\cdot}5} + 1{\cdot}5e^3 + 1{\cdot}75e^{3{\cdot}5} + e^4)$

(D) $\frac{1}{4}(e^2 + 2{\cdot}5e^{2{\cdot}5} + 3e^3 + 3{\cdot}5e^{3{\cdot}5} + 2e^4)$

x	1	1.25	1.5	1.75	2
xe^{2x}	e^2	$1.25e^{2.5}$	$1.5e^3$	$1.75e^{3.5}$	$2e^4$

$h = 0.25$

10. What is the nature and coordinates of the stationary point of the curve $y = \dfrac{\ln x}{x^3}$?

(A) A minimum turning point at $\left(e^{\frac{1}{3}}, \frac{1}{3e}\right)$.

(B) A maximum turning point at $\left(\frac{1}{3e}, e^{\frac{1}{3}}\right)$.

(C) A minimum turning point at $\left(\frac{1}{3e}, e^{\frac{1}{3}}\right)$.

(D) A maximum turning point at $\left(e^{\frac{1}{3}}, \frac{1}{3e}\right)$.

End of Section I

The paper continues in the next section

$u = \ln x \qquad v = x^3$

$u' = \frac{1}{x} \qquad v' = 3x^2$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{\frac{x^3}{x} - 3x^2 \ln x}{(x^3)^2}$$

$$= \frac{x^2 - 3x^2 \ln x}{x^6}$$

$$= \frac{x^2(1 - 3\ln x)}{x^6}$$

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$$y' = \frac{1 - 3\ln x}{x^4}$$

$1 - 3\ln x = 0$

$-3\ln x = -1$

$\ln x = \frac{1}{3}$

$e^{\ln x} = e^{\frac{1}{3}}$

$x = e^{\frac{1}{3}}$

$$y = \frac{\ln e^{\frac{1}{3}}}{(e^{\frac{1}{3}})^3}$$

$$= \frac{\frac{1}{3}}{e}$$

$$= \frac{1}{3e}$$

QUESTION ELEVEN (2 marks) **Marks**

Find the equation of the tangent to the curve $y = x^3 - x + 4$ at $x = 1$. $\boxed{2}$

$y' = 3x^2 - 1$

At $x = 1$ $y' = 2$ ✓

$x = 1$, $y = 1 - 1 + 4$ $(1, 4)$

$= 4$

$y - 4 = 2(x - 1)$

$= 2x - 2$ ✓

$\underline{y = 2x + 2}$

QUESTION TWELVE (4 marks) **Marks**

Differentiate:

(a) $y = \sqrt{x}$ $\boxed{1}$

$y = x^{\frac{1}{2}}$

$y' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ ✓

(b) $y = \cos 2x$ $\boxed{1}$

$y' = -2\sin 2x$ ✓

(c) $y = x^3 \ln x$ $\boxed{2}$

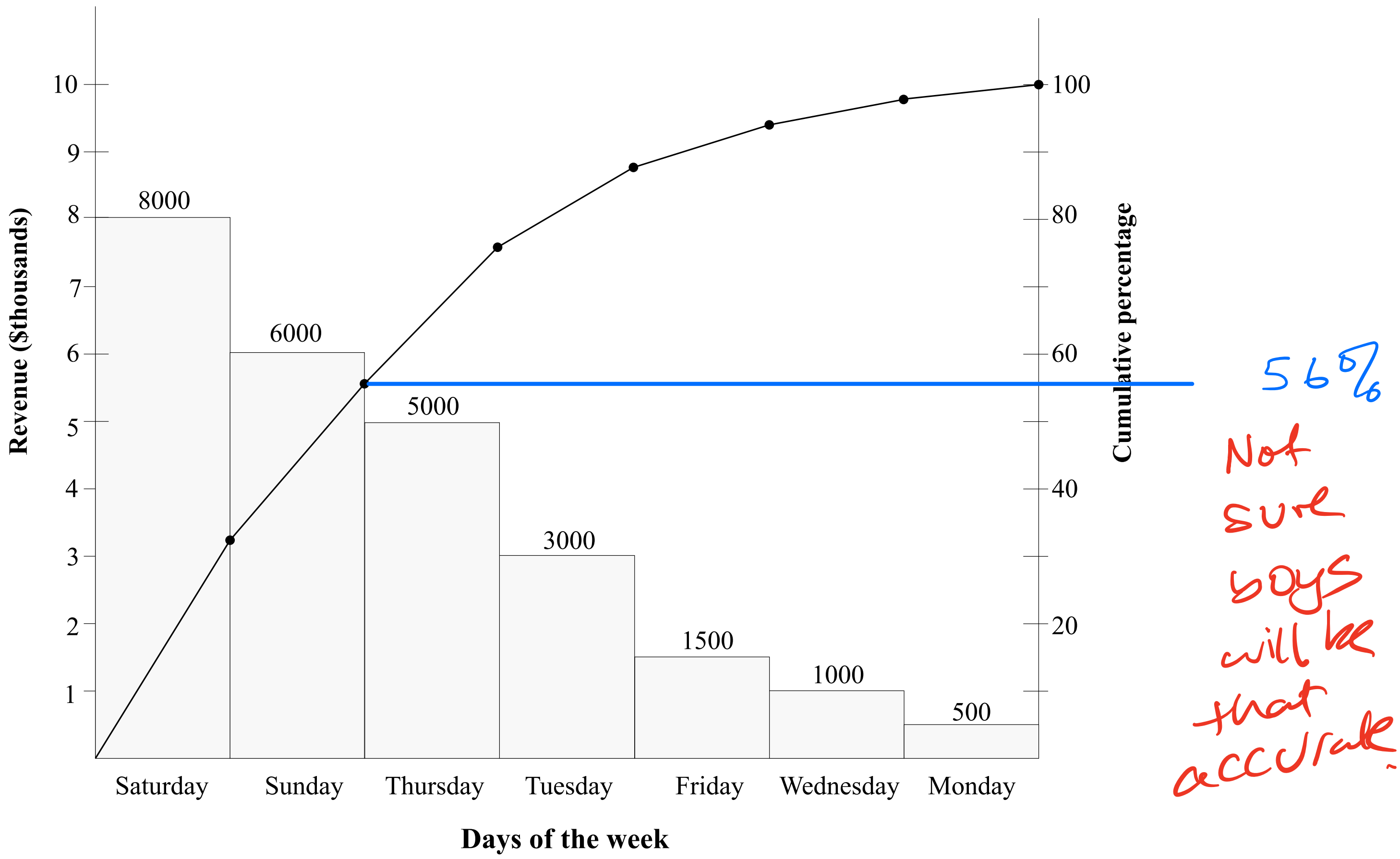
$y = uv$ $u = x^3$ $v = \ln x$

$y' = vu' + uv'$ $u' = 3x^2$ $v' = \frac{1}{x}$

$= 3x^2 \ln x + x^2$ ✓

✓

QUESTION THIRTEEN (3 marks) **Marks**



Not sure boys will be that accurate.

The diagram above shows a Pareto chart of the revenue that a bookshop made during a week.

(a) What percentage of the total revenue was made on the week days? $\boxed{2}$

Σsales = 25000 ✓

Sat + Sun = 14000

25000 − 14000 = 11000

$\frac{\cancel{11000}}{\cancel{25000}} \times \frac{\cancel{100}^{4}\%}{1} = 44\%$ ✓

(b) Suggest one valid action the manager may consider using the Pareto chart results. $\boxed{1}$

1. More staff on weekends.

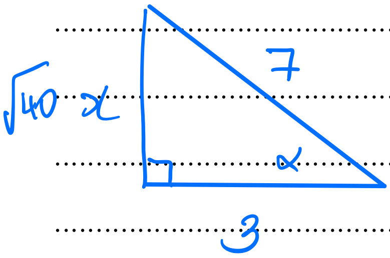
2. A sale early in the week to encourage customes. ✓

3. Close on a Monday.

4. Longer trading hours on weekend.

QUESTION FOURTEEN (2 marks) **Marks**

If $\cos\theta = -\frac{3}{7}$ and $\tan\theta$ is positive, find the value of $\sin\theta$. Leave your answer in simplified form. $\boxed{2}$



$x^2 + 3^2 = 7^2$

$x^2 = 49 - 9$

$x = \sqrt{40}$ ✓

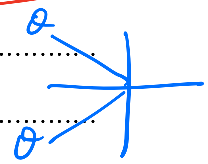
$\sin\alpha = \frac{\sqrt{40}}{7}$

$\cos\theta < 0$

but

$\tan\theta > 0$

Quadrant 3



$\sin\theta = -\frac{2\sqrt{10}}{7}$ ✓

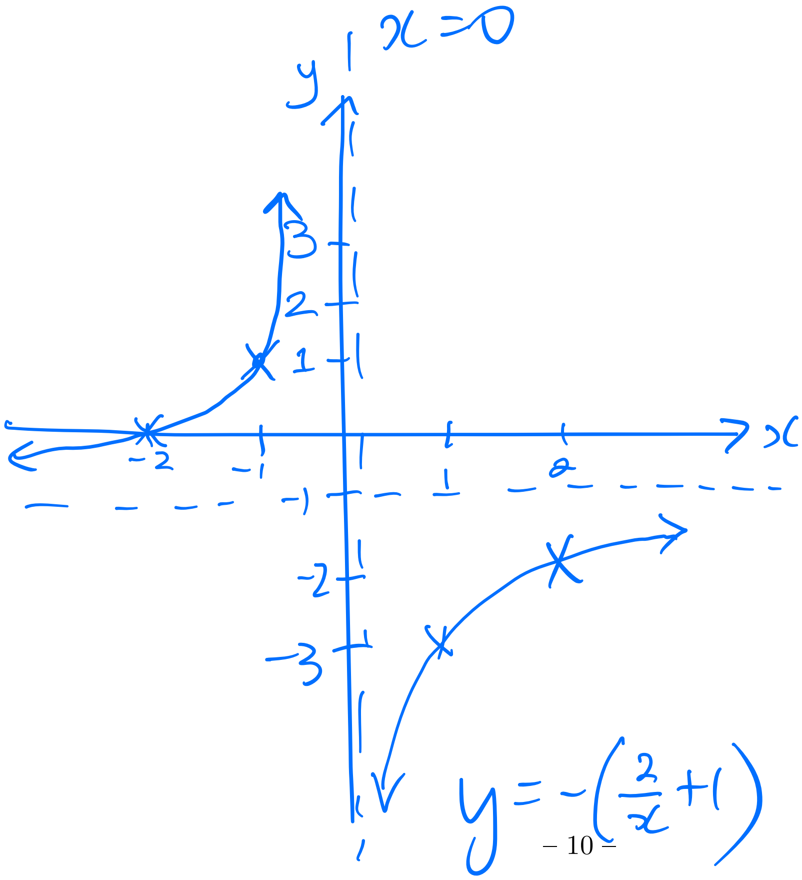
QUESTION FIFTEEN (4 marks) **Marks**

The graph of $y = \dfrac{2}{x}$ is translated upwards by 1 unit followed by a reflection in the x-axis.

(a) State the equation of the new graph. $\boxed{1}$

$y = -\left(\frac{2}{x} + 1\right)$ $y = -\frac{2}{x} - 1$ ✓

(b) Sketch the new graph. Clearly indicate any intercepts with the axes and any asymptotes. $\boxed{3}$



✓ shape

✓ intercepts

✓ asymptotes

QUESTION SIXTEEN (4 marks)

Marks

How many terms are there in the series $11 + 13 + 15 + \cdots$ if the sum is 375? 2

AP $a = 11$

$d = 2$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$375 = \frac{n}{2}\left(22 + (n-1)2\right)$$

$$750 = n\left(22 + 2n - 2\right)$$

$$= n\left(20 + 2n\right)$$

$$= 20n + 2n^2$$

$2n^2 + 20n - 750 = 0$

$n^2 + 10n - 375 = 0$

$(n - 15)(n + 25) = 0$

375

25 15

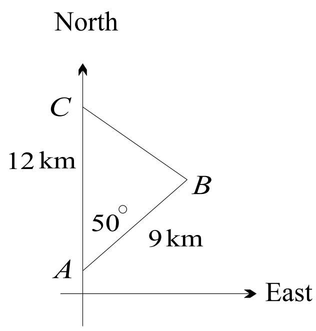
$n - 15 = 0$ $n + 25 = 0$

$n = 15$ $n = -25$

$n > 0$

Fifteen terms are needed to give a sum of 375.

QUESTION SEVENTEEN (3 marks) **Marks**



The diagram above shows three checkpoints A, B and C in an orienteering event. Checkpoints A and C are such that C is 12 km due north of A. One participant starts from A and walks in the direction of 050° T. After 9 km the participant arrives at checkpoint B.

(a) Find the distance BC. Give your answer correct to 3 significant figures. [1]

$BC^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \cos 50^\circ$

$= 86.157\ldots$

$BC \doteqdot 9.28$ km

(b) What is the true bearing of C from B? Give your answer correct to the nearest degree. [2]

Let $\angle ACB = \theta$

$\dfrac{9}{\sin\theta} = \dfrac{BC}{\sin 50^\circ}$

$\dfrac{\sin\theta}{9} = \dfrac{\sin 50^\circ}{BC}$

$\sin\theta = \dfrac{9 \sin 50^\circ}{BC}$ (0.7428)

$\theta \doteqdot 48^\circ$

Bearing $360 - 48^\circ = 312^\circ T$

check ambiguous case

$180 - 48^\circ$

$= 132^\circ$

$132 + 50 > 180^\circ$

θ is acute

OR

If use $\angle ABC = \alpha$

$\dfrac{\sin\alpha}{12} = \dfrac{\sin 50^\circ}{BC}$

$\sin\alpha = \dfrac{12 \sin 50}{BC}$ (0.9903)

$\alpha = 82^\circ$

Bearing $= 82 + 50 + 180 = 312^\circ T$.

QUESTION EIGHTEEN (2 marks) **Marks**

Solve the following equation for x in terms of a: 2

$$3\log_a x + 4 = 5\log_a x$$

$4 = 5\log_a x - 3\log_a x$

$4 = 2\log_a x$ ✓

$2 = \log_a x$

$x = a^2$ ✓

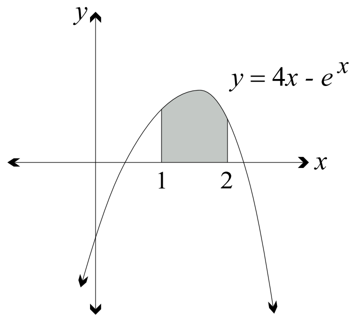
The paper continues on page 15.

QUESTION NINETEEN (3 marks) **Marks**

(a) Find $\int \frac{5}{x}\, dx$. 1

$5\ln|x| + C$

(b) 2



For the diagram above, find the exact area bounded by the curve, $y = 4x - e^x$ and the x-axis between $x = 1$ and $x = 2$.

$\int_1^2 (4x - e^x)\, dx = \left[\frac{4x^2}{2} - e^x\right]_1^2$

$= (8 - e^2) - (2 - e^1)$

$= (6 - e^2 + e)$ units^2

QUESTION TWENTY (5 marks) **Marks**

x	1	2	3	4	Sum
$P(X=x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	1
$x\,P(x)$	$\frac{3}{10}$	$\frac{8}{10}$	$\frac{6}{10}$	$\frac{4}{10}$	2.1 μ
$x^2 P(x)$	$\frac{3}{10}$	$\frac{16}{10}$	$\frac{18}{10}$	$\frac{16}{10}$	5.3 $E(x^2)$

The table above shows the probability distribution of a spinner for a board game. Let X be the outcome of the spinner.

(a) Find $P(X \le 3)$. [1]

$$\frac{3}{10}+\frac{4}{10}+\frac{2}{10} = \frac{9}{10}$$ ✓

(b) Find the expected value $E(X)$. [2]

$$E(x) = \sum x\,p(x)$$
$$= \frac{3}{10} + 2\times\frac{4}{10} + 3\times\frac{2}{10} + 4\times\frac{1}{10}$$ ✓
$$= \frac{3}{10}+\frac{8}{10}+\frac{6}{10}+\frac{4}{10}$$
$$= 2.1$$ ✓

(c) Find the variance $\mathrm{Var}(X)$. [2]

working above with table

$$\mathrm{Var}(X) = E(X^2) - \mu^2 \qquad E(X^2)=5.3$$ ✓
$$= 5.3 - (2.1)^2$$
$$= \underline{0.89}$$ ✓

QUESTION TWENTY-ONE (7 marks) **Marks**

Consider the curve $y = 4x^2 - 2x^3$.

(a) Find the stationary points of the curve $y = 4x^2 - 2x^3$. Determine their nature. $\boxed{3}$

$y' = 8x - 6x^2$
$= 2x(4 - 3x)$
$y'' = 8 - 12x$
$y' = 0 \quad 2x(4-3x) = 0$
$x = 0 \quad 4 - 3x = 0$
$-3x = -4$
$x = \frac{4}{3}$ ✓

When $x = 0$
$y'' = 8$ ✓
At $\begin{cases} x = 0 \\ y = 0 \end{cases}$ minimum turning point

When $x = \frac{4}{3}$
$y'' = 8 - \cancel{12}^{4} \times \frac{4}{\cancel{3}}$
$= -8$ ✓
At $\begin{cases} x = \frac{4}{3} \\ y = 4\left(\frac{4}{3}\right)^2 - 2\left(\frac{4}{3}\right)^2 = \frac{64}{27} \left(= 2\frac{10}{27}\right) \end{cases}$ maximum turning point.

(b) Show that there is an inflection point at $\left(\frac{2}{3}, \frac{32}{27}\right)$ on the curve. 1

$y'' = 8 - 12x$

$y'' = 0$

$8 - 12x = 0$

$-12x = -8$

$x = \frac{8 \div 4}{12 \div 4}$

$= \frac{3}{2}$

x	0	$\frac{3}{2}$	$\frac{4}{3}$
y''	8	0	-8
concavity	∪		∩

$\therefore x = \frac{2}{3}$ is an inflection point.

Must show concavity change. ✓

(c) For what interval is the curve $y = 4x^2 - 2x^3$ increasing? 1

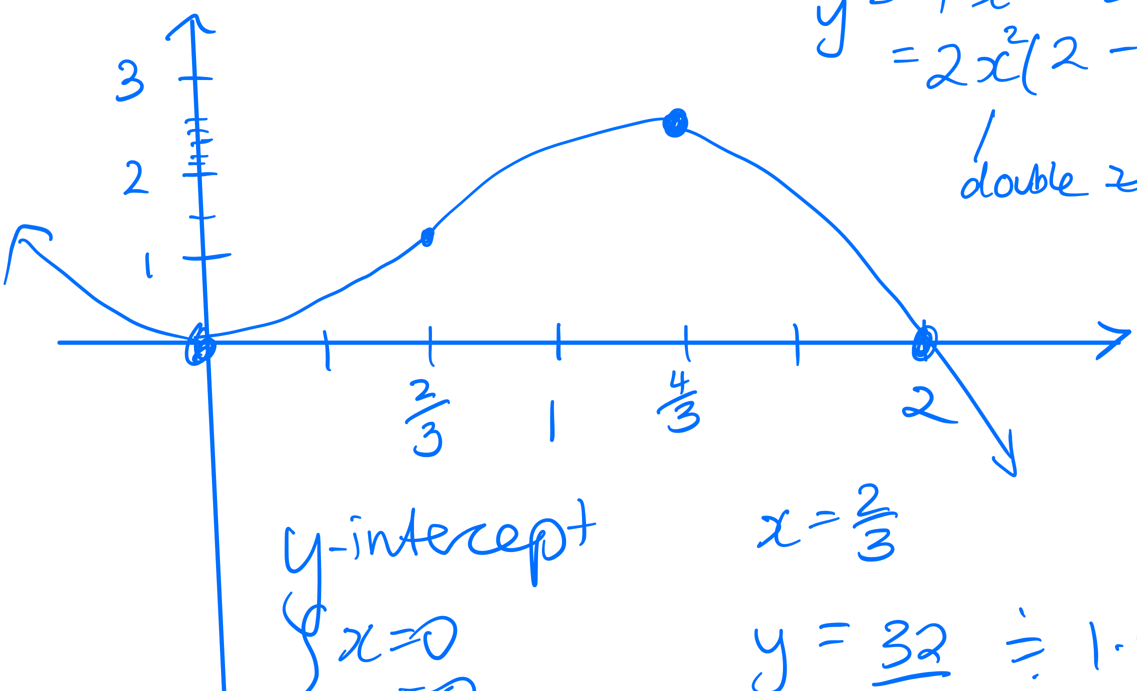
$0 < x < \frac{4}{3}$ ✓

(d) Hence, sketch the graph of the curve $y = 4x^2 - 2x^3$. Clearly label the stationary points, the point of inflection and any intercepts with the axes. 2

$y = 4x^2 - 2x^3$

$= 2x^2(2 - x)$

double zero. single zero



✓ shape

✓ intercepts

y-intercept

$\begin{cases} x = 0 \\ y = 0 \end{cases}$

$x = \frac{2}{3}$

$y = \frac{32}{27} \doteqdot 1.19.$

QUESTION TWENTY-TWO (4 marks) **Marks**

A particle moves along a straight line so that its displacement x metres to the right of a fixed point O is given by

$$x = 12\ln(t+2) - 2t + 5\,,$$

where the time t is measured in seconds.

(a) What is the initial position of the particle? Give your answer in exact form. 1

$t=0$ $\quad \underbrace{x = 12\ln 2 + 5}$ to the right

✓ sufficient.

(b) Find the expression for the velocity of the particle at time t. 1

$v = \frac{dx}{dt} = \frac{12}{t+2} - 2$ ✓

(c) Find the time when the particle is at rest. 1

$v=0 \quad \frac{12}{t+2} - 2 = 0$

$\frac{12}{t+2} = 2$

$12 = 2t + 4$

$2t = 8$ ✓

$\underline{t = 4s}$

(d) What happens to the acceleration eventually? 1

$v = 12(t+2)^{-1} - 2$ $\quad (t+2)^2 \geq 4$

$a = \frac{dv}{dt} = -12(t+2)^{-1}$ As $t \to \infty$ $\quad (t+2)^2 \to \infty$

$= \frac{-12}{(t+2)^2}$ $\quad a \to 0$ ✓

This is the halfway point of the whole paper.

Also accept at $t \to \infty$ $\quad v \to$ constant

$\therefore$ zero acceleration.

QUESTION TWENTY-THREE (2 marks)

Marks
2

A curve $y = f(x)$ passes through $\left(\frac{\pi}{2}, \frac{-\pi}{2}\right)$ and has the gradient function

$$\frac{dy}{dx} = 4\cos 2x + 1\,.$$

Find the equation of the curve.

$\int (4\cos 2x + 1)\,dx$

$= 4 \times \frac{1}{2}\sin 2x + x + C$ ✓

$y = 2\sin 2x + x + C$

At $\left(\frac{\pi}{2}, -\frac{\pi}{2}\right)$ $\quad -\frac{\pi}{2} = 2\sin \cancel{2}\frac{\pi}{\cancel{2}} + \frac{\pi}{2} + C$

$-\frac{\pi}{2} = 0 + \frac{\pi}{2} + C$

$C = -\pi$

$\therefore\ y = 2\sin 2x + x - \pi$. ✓

QUESTION TWENTY-FOUR (3 marks)

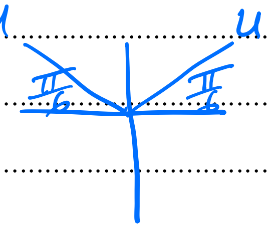
Marks
3

Solve the equation $2\sin 2x = 1$ for $0 \le x \le 2\pi$.

Let $u = 2x$ $\qquad 0 \le 2x \le 4\pi$

$2\sin u = 1$ $\qquad 0 \le u \le 4\pi$ ✓

$\sin u = \frac{1}{2}$



RA $= \frac{\pi}{6}$ ✓

$u = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$x = \frac{u}{2}$

✓

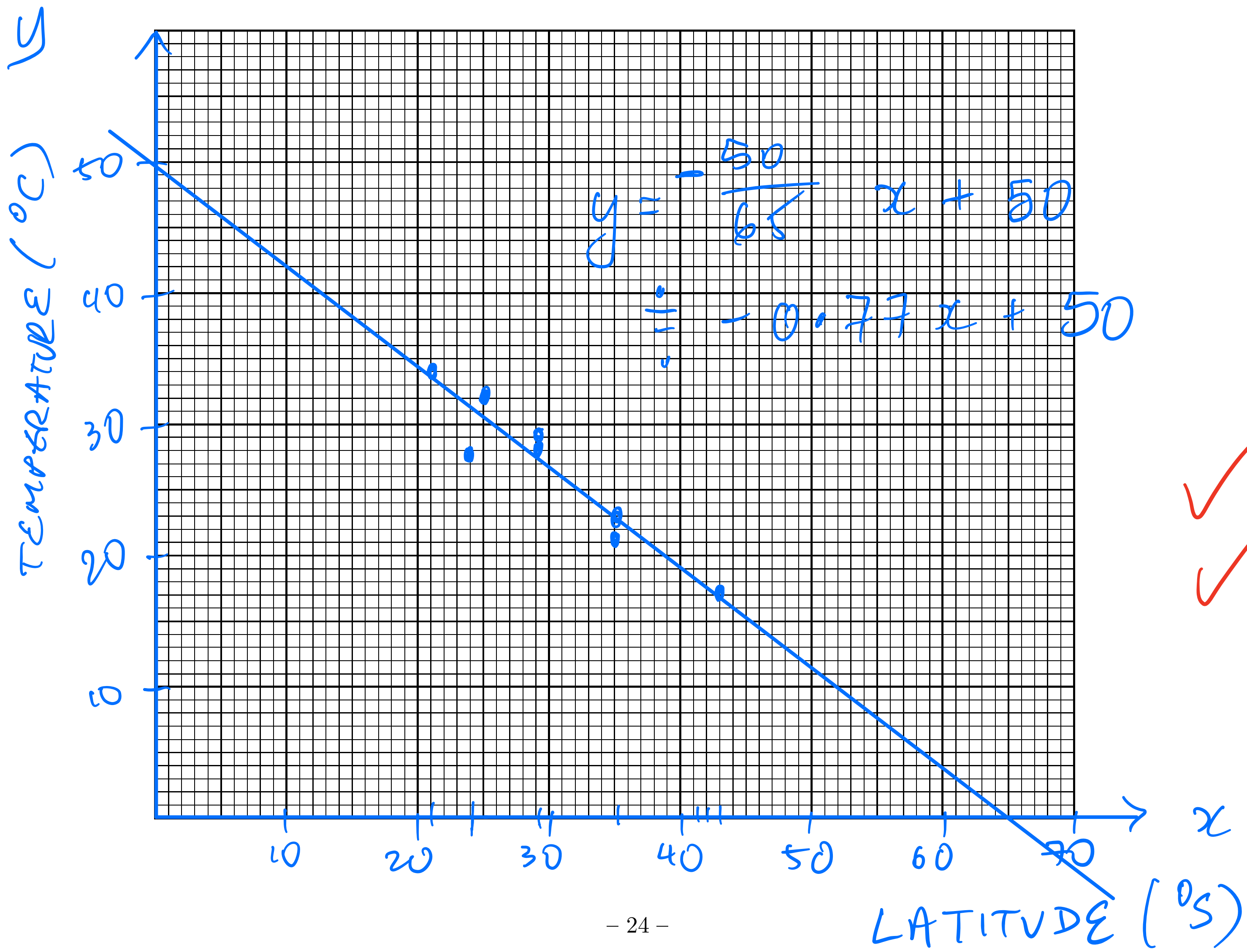
$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{2}$

QUESTION TWENTY-FIVE (8 marks) **Marks**

The table below shows the latitude (degrees south of the equator) and the temperature on a particular day (°C) of eight locations in Australia.

Location	Latitude	Temperature
	°South	°C
Alice Springs	24	28
Byron Bay	29	28
Carnavon	25	32
Geraldton	29	29
Hobart	43	17
Mt. Isa	21	34
Port Lincoln	35	21
Wagga Wagga	35	23

(a) Draw a scatter plot to show any potential relationship between latitude and temperature in the southern hemisphere. Let the horizontal axis be latitude and the vertical axis be temperature. 2



(b) Describe the relationship between latitude and temperature observed in the scatter plot. $\boxed{1}$

As latitude south increases (away from equator) temperature is decreasing.

(c) By eye, estimate and draw in the line of best fit on your scatter plot in part (a). Hence determine the y-intercept and the gradient for your line. Give your answers to 2 decimal places. $\boxed{2}$

y-intercept +50

gradient $= -\frac{50}{65}$ $y = -0.77x + 50$

$\doteqdot -0.77$

(d) (i) Using your calculator, find Pearson's correlation coefficient r for this data. Give your answer to 4 significant figures. $\boxed{1}$

$r \doteqdot -0.9571$

(ii) Comment on the significance of the value r for this set of data. $\boxed{1}$

r close to −1 Strong negative correlation

(e) Using your calculator, find the equation of the line of regression. Give the y-intercept and the gradient for the line to 3 significant figures. $\boxed{1}$

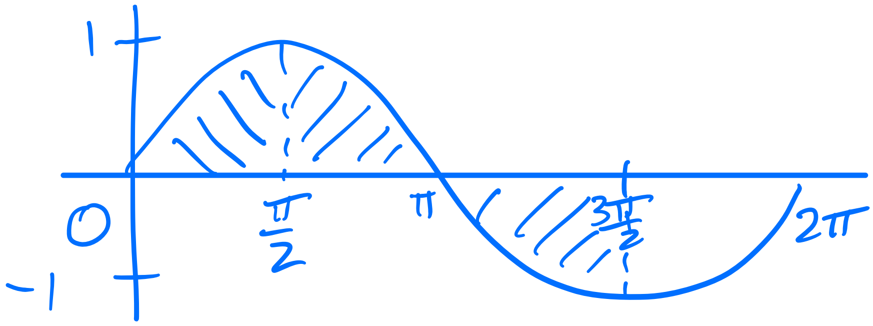
$y = A + Bx$

$A \doteqdot 49.5$ $B \doteqdot -0.762$

$y = -0.762x + 49.5$

QUESTION TWENTY-SIX (4 marks) **Marks**

(a) Graph $y = \sin x$ for $0 \leq x \leq 2\pi$. 2



✓ intercepts

✓ shape

(do not mark shading)

(b) Shade the regions bounded by the curve $y = \sin x$, the x-axis and between $x = 0$ and $x = \dfrac{3\pi}{2}$. Calculate the total area of these regions. 2

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = \left[-\cos x\right]_0^{\frac{\pi}{2}}$$ ✓

$$= -\cos\frac{\pi}{2} - (-\cos 0)$$

$$= 0 + 1$$

$$= 1$$

Total Area $= 3$ units^2 ✓

QUESTION TWENTY-SEVEN (3 marks) **Marks**

Evaluate:

(a) $\frac{1}{2}\int \frac{2x}{x^2 - 5}\, dx$ 1

$= \frac{1}{2} \ln \left| x^2 - 5 \right| + c$ ✓

must have absolute value signs.

(b) $\displaystyle\int_1^5 \frac{x^2 + 3}{x}\, dx$ 2

$= \int_1^5 \left(x + \frac{3}{x} \right) dx$

$= \left[\frac{x^2}{2} + 3\ln x \right]_1^5$ ✓

$= \frac{25}{2} + 3\ln 5 - \left(\frac{1}{2} + 3\ln 1 \right)$

$= 12 + 3\ln 5$ ✓

QUESTION TWENTY-EIGHT (4 marks) **Marks**

Hugo has three years to save $25 000 for a holiday.

(a) Hugo deposits a single lump sum into an account paying 8% p.a. interest compounded every 6 months. What lump sum is needed to ensure he can afford his holiday in three years time? Give your answer to the nearest dollar. 2

$PV \times (1.04)^6 = 25000$ $R = 4\%$ per 6 months

$PV \doteqdot \$19758$ $n = 3 \times 2$ months

(b) 2

Periods n	Interest rate per period 3%	4%	5%	6%	8%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0300	2.0400	2.0500	2.0600	2.0800	2.1200
3	3.0909	3.1216	3.1525	3.1836	3.2464	3.3744
4	4.1836	4.2465	4.3101	4.3746	4.5061	4.7793
5	5.3091	5.4163	5.5256	5.6371	5.8666	6.3528
6	6.4684	6.6330	6.8019	6.9753	7.3359	8.1152
7	7.6625	7.8983	8.1420	8.3938	8.9228	10.0890
8	8.8923	9.2142	9.5491	9.8975	10.6366	12.2997
9	10.1591	10.5828	11.0266	11.4913	12.4876	14.7757
10	11.4639	12.0061	12.5779	13.1808	14.4866	17.5487
11	12.8078	13.4864	14.2068	14.9716	16.6455	20.6546
12	14.1920	15.0258	15.9171	16.8699	18.9771	24.1331

Hugo instead decides to make regular deposits to an annuity to save for his holiday. He deposits $1800 at the end of each quarter over 3 years at 12% p.a. interest compounded quarterly.

Use the future value table above to determine if Hugo will have enough money to take his holiday. Show working to explain your answer.

R: $\frac{12\%}{4}$ pa compound quarterly.

3% per qtr

$n = 3 \times 4$

$= 12$ qtr.

$FV = 1800 \times 14.1920$

$= \$25546$ ∴ Hugo has enough money for his holiday.

some appropriate working

QUESTION TWENTY-NINE (6 marks) **Marks**

A closed cylindrical tank has a circular base of radius r metres and a height of h metres. It can hold $160\pi\,\text{m}^3$ of wheat. The material for the flat circular top and bottom costs \$10 per square metre and the material for the curved surface costs \$8 per square metre.

(a) Show that the height of the tank is $h = \dfrac{160}{r^2}$. $\boxed{1}$

$V = \pi r^2 h$

$160\pi = \pi r^2 h$

$h = \dfrac{160}{r^2}$

(b) Find the dimensions of the tank that minimise the cost of construction. $\boxed{5}$

Cost $(A_{top} + A_{bottom}) = 2 \times \pi r^2 \times \$10 = \$20\pi r^2$

Area of curved surface $= 2\pi r h$

$= 2\pi r \times \dfrac{160}{r^2}$

$= \dfrac{320\pi \cancel{r}}{r^{\cancel{2}}}$

$\text{Cost}_{cs} = \dfrac{320\pi}{r} \times \$8$

$= \$\dfrac{2560\pi}{r}$

Total $C = 20\pi r^2 + 2560\pi r^{-1}$

$\dfrac{dC}{dr} = 40\pi r - 2560\pi r^{-2}$

$C'' = 40\pi + \dfrac{5120\pi}{r^3}$

$\dfrac{dc}{dr} = 0$ $\quad \dfrac{2560\cancel{\pi}}{r^2} = 40\cancel{\pi} r$

$\dfrac{256\cancel{0}}{4\cancel{0}} = r^3$

$r^3 = 64$

$r = 4$

when $r = 4$

$C'' = 40\pi + \dfrac{5120\pi}{64}$

> 0

Minimum when

$r = 4$ m

$h = \dfrac{160}{4^2}$

$= 10$m

dimensions

QUESTION THIRTY (3 marks) **Marks**

Prove the identity $(\sec\theta + \tan\theta)^2 = \dfrac{1+\sin\theta}{1-\sin\theta}$. $\boxed{3}$

$$LHS = (\sec\theta + \tan\theta)^2$$

$$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)^2 \checkmark$$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta} \checkmark$$

$$= \frac{(1+\sin\theta)^{\cancel{2}}}{(1-\sin\theta)\cancel{(1+\sin\theta)}} \checkmark$$

$$= \frac{1+\sin\theta}{1-\sin\theta}$$

QUESTION THIRTY-ONE (7 marks) **Marks**

Tom invented a dice game to play on his own. He throws a pair of six-sided dice repeatedly until the difference between the dice is 2 or 3.
If the difference is 2, Tom wins and the game ends.
If the difference is 3, Tom loses and the game ends.
If the difference is any other number, he continues to throw until the difference is a 2 or 3.

(a) Show that the probability that Tom wins on his first throw of the dice is $\frac{2}{9}$. [2]

Die 2						
6	5	4	3	2	1	0
5	4	3	2	1	0	1
4	3	2	1	0	1	2
3	2	1	0	1	2	3
2	1	0	1	2	3	4
1	0	1	2	3	4	5
	1	2	3	4	5	6

Die 1

✓ Some appropriate working.

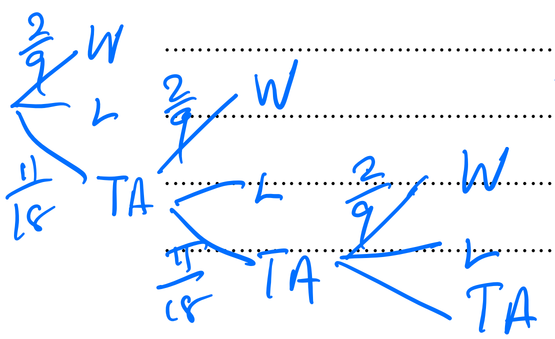
$$P(\text{wins first throw}) = \frac{8}{36} \div 4 \div 4 = \frac{2}{9}$$ ✓

(b) Calculate the probability that the game continues to a second throw. [1]

$$P(2 \text{ or } 3) = \frac{8+6}{36} = \frac{14}{36}$$

$$P(\overline{2 \text{ or } 3}) = 1 - \frac{14}{36} = \frac{11}{18}$$ ✓

(c) What is the probability that Tom wins in one of the first three throws? Leave your answer in unsimplified form. [2]



Win Loss

TA = Throw Again

$$\underbrace{\frac{2}{9} + \frac{11}{18} \times \frac{2}{9}}_{✓} + \frac{11}{18} \times \frac{11}{18} \times \frac{2}{9} = \frac{643}{1458}$$ ✓ (simplified)

(d) Calculate the probability that Tom wins the game. [2]

$$S_\infty = \frac{a}{1-r} = \frac{\frac{2}{9}}{1-\frac{11}{18}} = \frac{2}{9} \times \frac{18}{7} = \frac{4}{7}.$$ ✓

$a = \frac{2}{9}$
$r = \frac{11}{18}$

identifies GP. ✓

QUESTION THIRTY-TWO (6 marks) **Marks**

On the 1st January 2021, Amanda invested \$7000 into a bank account that paid interest at a rate of 6% p.a. compounded annually. Amanda decided to add \$700 to her account on the 1st January each year, beginning in 2022.

Let A_n be the amount in the account on the 1st January after n years, after interest and her deposit has been paid.

(a) Show that $A_2 = A_1 \times 1{\cdot}06 + 700$. **1**

$A_0 = 7000$ (optional)

$A_1 = 7000 \times 1{\cdot}06 + 700$

$A_2 = A_1 + 0{\cdot}06 A_1 + 700$

$= A_1(1{\cdot}06) + 700$

$= (7000 \times 1{\cdot}06 + 700)(1{\cdot}06) + 700$

$= 7000 \times 1{\cdot}06^2 + 700 \times 1{\cdot}06 + 700$

✓ must see.

(b) Hence, determine how much was in her account on 1st January 2031, after interest and her deposit has been paid. Give your answer to the nearest dollar. **3**

$A_3 = 7000 \times 1{\cdot}06^3 + 700 \times 1{\cdot}06^2 + 700 \times 1{\cdot}06 + 700$

✓ must generate the pattern & identify the GP.

$A_n = 7000 \times 1{\cdot}06^n + \underbrace{700 \times 1{\cdot}06^{n-1} + \cdots + 700 \times 1{\cdot}06^2 + 700 \times 1{\cdot}06 + 700}$

this is a GP

$a = 700$

$r = 1{\cdot}06$

n terms

$A_n = 7000 \times 1{\cdot}06^n + \dfrac{700(1{\cdot}06^n - 1)}{0{\cdot}06}$ ✓

Just this is not enough for full marks.

$A_{10} = 7000 \times 1{\cdot}06^{10} + \dfrac{700(1{\cdot}06^{10} - 1)}{0{\cdot}06}$

$\doteqdot \$21\,762$ (nearest dollar) ✓

(c) Amanda's friend, Bard, invested \$7000 into an account in a different bank on the 1st January 2021 and made no further payments. On 1st January 2031, Bard's balance was \$23 417. $\boxed{2}$

Calculate the annual rate of compound interest paid on Bard's account. Give your answer to 4 significant figures.

$$7000\left(1+\frac{R}{100}\right)^{10} = 23\,417$$

$$\left(1+\frac{R}{100}\right)^{10} = \frac{23417}{7000}$$

$$1+\frac{R}{100} = \sqrt[10]{\frac{23417}{7000}}$$

$$\frac{R}{100} = \sqrt[10]{\frac{23417}{7000}} - 1$$

$$R = 12 \cdot 83\,\%$$

✓ one line of appropriate working.

✓

The paper continues on the next page.

QUESTION THIRTY-THREE (3 marks) **Marks**

Find the area bounded by the curves $y = \sqrt{4x + 8}$ and $5y - 2x = 12$. $\boxed{3}$

$y = \sqrt{4x+8}$
$= 2\sqrt{x+2}$

$5y = 2x + 12$

$5 \times \sqrt{4(x+2)} = 2x + 12$

$10\sqrt{x+2} = 2x + 12$

square both sides:

$100(x+2) = (2x+12)^2$

$100x + 200 = 4x^2 + 48x + 144$

$4x^2 - 52x - 56 = 0$

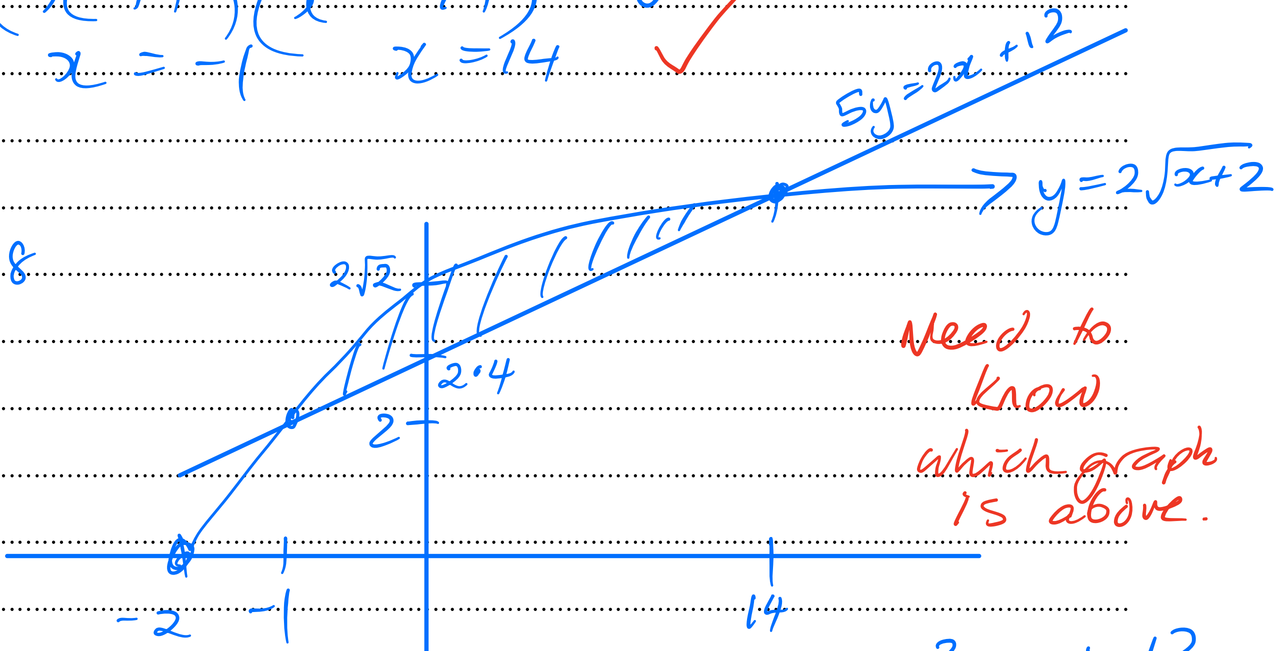
$\div 4$

$x^2 - 13x - 14 = 0$

$(x+1)(x-14) = 0$

$x = -1 \quad x = 14$ ✓

$2\sqrt{2} \approx 2.8$



Need to know which graph is above.

$x = -1 \quad y = \sqrt{-4+8}$
$= 2$

$x = 14 \quad y = \sqrt{64}$
$= 8$

$y = \frac{2}{5}x + \frac{12}{5}$

$y = \frac{2}{5}x + \underline{2.4}$

$$A = \int_{-1}^{14} \sqrt{4x+8} - \left(\frac{2}{5}x + \frac{12}{5}\right) dx$$

$$= \int_{-1}^{14} \left((4x+8)^{\frac{1}{2}} - \frac{2}{5}x - \frac{12}{5} \right) dx$$

$$= \left[\frac{2(4x+8)^{\frac{3}{2}}}{4 \times 3} - \frac{2x^2}{5 \times 2} - \frac{12x}{5} \right]_{-1}^{14}$$

$$= \left[\frac{(4x+8)^{\frac{3}{2}}}{6} - \frac{x^2}{5} - \frac{12x}{5} \right]_{-1}^{14}$$

$$= \left(\frac{(4 \times 14 + 8)^{\frac{3}{2}}}{6} - \frac{14^2}{5} - \frac{12 \times 14}{5} \right) - \left(\frac{(4 \times -1 + 8)^{\frac{3}{2}}}{6} - \frac{(-1)^2}{5} - \frac{12 \times (-1)}{5} \right)$$

$$= \left(\frac{64^{\frac{3}{2}}}{6} - \frac{26 \times 14}{5} \right) - \left(\frac{4^{\frac{3}{2}}}{6} - \frac{1}{5} + \frac{12}{5} \right)$$

$$= \left(\frac{8 \times 8 \times 8}{6} - \frac{364}{5} \right) - \left(\frac{2 \times 2 \times 2}{6} + \frac{11}{5} \right)$$

$$= \frac{256}{3} - \frac{364}{5} - \frac{4}{3} - \frac{11}{5}$$

$$= \frac{252}{3} - \frac{375}{5}$$

$$= 84 - 75$$

$$= 9 \text{ units}^2$$

END OF PAPER